PHYS 320 ANALYTICAL MECHANICS

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Fall 2018

Suggested Final Exam time change:
Wednesday, 12 Dec, 6:30 – 8:20 pm
[from Thursday, 13 Dec, 3:30 – 5:30 pm]

Lagrangian Mechanics

Lagrangian function:

$$L \equiv T(q_j, \dot{q}_j, t) - U(q_j, t)$$

Euler-Lagrange equations of motion:

$$\frac{\partial L}{\partial q_j} \equiv \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j}, \quad j = 1, 2, 3, \ldots, s$$

Lagrangian Mechanics: single particle in 3D

Lagrangian: $L \equiv T - U = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U(x,y,z)$

Euler-Lagrange equations of motion:

$$\left.\begin{aligned} &\frac{\partial L}{\partial x} = -\frac{\partial U}{\partial x} = F_x \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = \frac{d}{dt}(m\dot{x}) = m\ddot{x} \\ &\frac{\partial L}{\partial y} = -\frac{\partial U}{\partial y} = F_y \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) = \frac{d}{dt}(m\dot{y}) = m\ddot{y} \\ &\frac{\partial L}{\partial z} = -\frac{\partial U}{\partial z} = F_z \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{z}} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}}\right) = \frac{d}{dt}(m\dot{z}) = m\ddot{z} \end{aligned}\right\} \quad \vec{F} = -\vec{\nabla}U = m\vec{a}$$

Lagrangian Mechanics:

$$\left.\begin{aligned} &\frac{\partial L}{\partial q_j} \equiv \text{ generalized force, jth component} \\ &\frac{\partial L}{\partial \dot{q}_j} = \text{ generalized momentum, jth component} \end{aligned}\right\} \quad \frac{\partial L}{\partial q_j} \equiv \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j}$$

generalized force
= rate of change of
generalized momentum